WHY IS IT NECESSARY TO KNOW THEOREMS ?

WHAT ARE THEOREMS GOOD FOR ANYWAY ?

Theorems are the real shortcuts of math. They allow you to reduce the amount of work by making connections between conditions and conclusions. If you can show that all the conditions of a theorem are met in a certain problem, then you can immediately jump to the conclusion of the theorem without any additional work. (It's like a ladder in the game of Chutes & Ladders, also known as Snakes & Ladders: if you can land on a square with a ladder on it, you can climb up the ladder directly, without going through all the squares between the bottom of the ladder and the top of the ladder.)

Here's a simple example which shows how much work can be saved:

You're given the number 211, and you want to prove it is prime.

Without any theorems, you would have to divide 211 by every prime number from 2 until just before 211.

$$\frac{211}{2}, \frac{211}{3}, \frac{211}{5}, \frac{211}{7}, \frac{211}{11}, \frac{211}{13}, \ \ldots \ \frac{211}{193}, \frac{211}{197}, \frac{211}{199}$$

That means you would have to divide 211 by 46 different prime numbers.

However, it turns out that there's a theorem that says:

If a positive integer has no factor (other than 1) that is less than or equal to its square root, then that integer is prime.

That means that you only need to try dividing 211 by the prime numbers less than or equal to $\sqrt{211} \approx 14.5$.

$$\frac{211}{2}, \frac{211}{3}, \frac{211}{5}, \frac{211}{7}, \frac{211}{11}, \frac{211}{13}$$

If none of those 6 prime numbers is a factor of 211,
then you've shown that the condition of the theorem is true,
and you can immediately jump to the conclusion of the theorem,
and say that 211 is prime.

In other words, if you know the theorem and you know how to use it,
then you've just saved yourself from doing 40 unnecessary divisions.

Your theorem-based shortcut saved you $\frac{40}{46} \times 100\% = 87\%$ of the work.

(And you probably saved more work than that, because you only had to divide by the smallest prime numbers, and none of the larger ones.)

Of course, **in order to use the theorem,**
you actually have to know there's such a theorem,
and you have to know what it says exactly,
to avoid using the theorem the wrong way, and drawing a wrong conclusion.
(The phrase "jumping to conclusions" refers to drawing a conclusion without checking that the conditions are met. Usually, the conclusion drawn is actually wrong.)

So, if you've been looking for shortcuts, they've been in front of you the entire time. The shortcuts were the theorems.